Binomial Theorem

Question1

The constant term in the expansion of $\left(1+\frac{1}{x}\right)^{20}\left(30x(1+x)^{29}+(1+x)^{30}\right)$ is

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Options:

A.

$^{50}C_{20}+30\cdot{}^{50}C_{29}$

B.

$^{50}C_{19}+30\cdot{}^{49}C_{19}$

C.

$^{50}C_{20}+30\cdot{}^{49}C_{20}$

D.

$^{50}C_{20}+30\cdot{}^{49}C_{19}$

Answer: D

Solution:

Let the given expression,

$E=\left(1+\frac{1}{x}\right)^{20}\left[30x(1+x)^{29}+(1+x)^{30}\right]$

Here, $30x(1+x)^{29}+(1+x)^{30}$

$$
\begin{aligned}
&= (1+x)^{29}[30x + (1+x)] \\
&= (1+x)^{29}(31x+1)
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad E &= \left(1+\frac{1}{x}\right)^{20}(1+x)^{29}(31x+1) \\
&= \frac{(x+1)^{20}}{x^{20}}(1+x)^{29}(31x+1) \\
&= \frac{(x+1)^{49}}{x^{20}}(31x+1) \\
&= (x+1)^{49}\left(31x^{-19}+x^{-20}\right) \\
&= 31x^{-19}(x+1)^{49} + x^{-20}(x+1)^{49}
\end{aligned}
$$

For constant term x^0

$\therefore$ Constant term in $31x^{-19}(x+1)^{49}$

i.e., $x^{-19} \cdot x^k = x^0 \Rightarrow k = 19$

$\therefore$ Constant term $= 31^{49}C_{19}$

Constant term in $x^{-20}(x+1)^{49}$

i.e., $x^{-20} \cdot x^k = x^0 \Rightarrow k = 20$

$\therefore$ Constant term $= 1 \times {}^{49}C_{20}$

$\therefore$ Sum of constant term

$$
\begin{aligned}
&= 31^{49}C_{19} + {}^{49}C_{20} \\
&= 30^{49}C_{19} + {}^{49}C_{19} + {}^{49}C_{20} \\
&= 30 \cdot {}^{49}C_{19} + {}^{50}C_{20}
\end{aligned}
$$

Question2

When $|x| > 3$, then coefficient of $\frac{1}{x^n}$ in the expansion of $x^{3/2}(3+x)^{1/2}$ is

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Options:

A.

$(-1)^n \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n n!} 3^n$

B.

$(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+2}(n+2)!} 3^{n+2}$

C.

$(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n n!} 3^{n+1}$

D.

$(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+3}(n+2)!} 3^{n+1}$

Answer: B

Solution:

We have, $x^{3/2}(3+x)^{1/2}$

$$= x^{3/2} x^{1/2}\left(1+\frac{3}{x}\right)^{1/2}$$
$$= x^2\left(1+\frac{3}{x}\right)^{1/2}$$

Now, coefficient of general term of $\left(1+\frac{3}{x}\right)^{1/2}$ is

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)\ldots\left(\frac{1}{2}-k+1\right)}{k!}$$
$$\frac{(-1)^{k+1} 1 \cdot 3 \cdot 5 \ldots (2k-3)}{2kk!}$$

On multiply by x^2, then

$x^2\left(1+\frac{3}{x}\right)^{1/2} = \sum_{k=0}^{\infty} {}^{1/2}C_k 3^k x^{2-k}$

For coefficient of $\frac{1}{x^n}, 2-k=-n$

$\Rightarrow \quad k=n+2$

Coefficient of $\frac{1}{x^n}$ is

$$\frac{(-1)^{n+3} 1 \cdot 3 \cdot 5 \ldots (2(n+2)-3) 3^{n+2}}{2^{n+2}(n+2)!}$$
$$= \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+1}(n+2)!} 3^{n+2}$$

Question3

If the coefficient of 3rd term from the beginning in the expansion of $\left(ax^2-\frac{8}{bx}\right)^9$ is equal to the coefficient of 3rd term from the end in the expansion of $\left(ax-\frac{2}{bx^2}\right)^9$, then the relation between a and b is

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Options:

A.

$ab=-1$

B.

$ab=1$

C.

$a^5b^5=-2$

D.

$a^5b^5=2$

Answer: C

Solution:

Given, coefficient of 3rd term from beginning in the expansion $\left(ax^2-\frac{8}{bx}\right)^9$ = Coefficient of 3rd term from end in the expansion $\left(ax-\frac{2}{bx^2}\right)^9$

For $\left(ax^2-\frac{8}{bx}\right)^9$

$$\begin{aligned} T_3 &= {}^9C_2\left(ax^2\right)^7\left(\frac{-8}{bx}\right)^2 \\ &= \frac{9!}{2!7!}a^7x^{14}\cdot\frac{64}{b^2x^2} \\ &= \frac{36\cdot a^7}{b^2}\cdot x^{12}\cdot 64 \end{aligned}$$

So, its coefficient is $\frac{36\cdot 64a^7}{b^2}$

For $\left(ax-\frac{2}{bx^2}\right)^9$

$$T_8 = {}^9C_7(ax)^2\left(\frac{-2}{bx^2}\right)^7$$
$$= \frac{9!}{2!7!}a^2x^2 \cdot \frac{-128}{b^7x^{14}}$$

So, if coefficient is

$= -36 \times 128\frac{a^2}{b^7}$

Now, $36 \cdot \frac{64a^7}{b^2} = -36 \cdot 128\frac{a^2}{b^7}$

$\Rightarrow \quad 64 \cdot a^5b^5 = -128$

$\Rightarrow \quad a^5b^5 = -2$

Question4

If the expression $5^{2n} - 48n + k$ is divisible by 24 for all $n \in N$, then the least positive integral value of k is

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Options:

A.

47

B.

48

C.

24

D.

23

Answer: D

Solution:

Given, $5^{2n} - 48n + k$ is divisible by 24

Let $\quad E(n) = 5^{2n} - 48n + k$

and $\quad E(n) \equiv 0(\bmod 24), \forall n \in N$

$\Rightarrow \quad 5^{2n} - 48n + k \equiv 0(\bmod 24)$

$\Rightarrow \quad k = 48n - 5^{2n}(\bmod 24)$

For $n = 1$,

$$\begin{aligned} k &= 48(1) - 5^{2 \times 1}(\bmod 24) \\ &= 48 - 25(\bmod 24) \\ &= 23(\bmod 24) \end{aligned}$$

For $n = 2$

$k = 48(2) - 5^{1}(\bmod 24)$

$$\begin{aligned} &= 96 - 625(\bmod 24) \\ &= 529(\bmod 24) = 23(\bmod 24) \end{aligned}$$

Thus, divisibility for 24 to hold for all n, we must have $k \equiv 23(\bmod 24)$

Thus, the least positive integral value of k is 23 .

Question5

If $X \sim B(7, P)$ is a binomial variate and $P(X = 3) = P(X = 5)$, then $P =$

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Options:

A.

$\frac{5-\sqrt{10}}{3}$

B.

$\frac{\sqrt{10}-2}{3}$

C.

$\frac{5-\sqrt{15}}{2}$

D.

$\frac{\sqrt{15}-3}{2}$

Answer: C

Solution:

Given $X \sim B(7, P)$ is a binomial variate and $P(X=3)=P(X=5)$

Using the binomial probability formula

$P(X=K)=\binom{n}{K} P^K(1-P)^{n-K}$

Here $n=7$,

So, $P(X=3)=\binom{7}{3} P^3(1-P)^4$

And, $P(X=5)=\binom{7}{5} P^5(1-P)^2$

$$
\begin{aligned}
& \therefore\binom{7}{3} P^3(1-P)^4=\binom{7}{5} P^5(1-P)^2 \\
& \Rightarrow 35 P^3(1-P)^4=21 P^5(1-P)^2 \\
& \Rightarrow 35(1-P)^2=21 P^2, \text { where } P \neq 0, P \neq 1 \\
& \Rightarrow 5(1-P)^2=3 P^2 \\
& \Rightarrow 5\left(1-2 P+P^2\right)=3 P^2 \\
& \Rightarrow 5-10 P+5 P^2-3 P^2=0 \\
& \Rightarrow 2 P^2-10 P+5=0 \\
& \Rightarrow P=\frac{-(-10) \pm \sqrt{100-4 \times 2 \times 5}}{2 \times 2} \\
& \quad=\frac{10 \pm \sqrt{100-40}}{4} \\
& \quad=\frac{10 \pm 2 \sqrt{15}}{4}=\frac{5 \pm \sqrt{15}}{2}
\end{aligned}
$$

But $P=\frac{5+\sqrt{15}}{2}>1$, So it can't be a valid probability.

$\therefore P=\frac{5-\sqrt{15}}{2}$

Question6

The coefficient of $x y^2 z^3$ in the expansion of $(x-2 y+3 z)^3$ is

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Options:

A. 6480

B. 3240

C. 1620

D. 810

Answer: A

Solution:

$(x-2y+3z)^6$

By multinomial theorem,

$$\Sigma \frac{6!}{p!q!r!}(x)^p(-2y)^q(3z)^r$$

$$\Rightarrow \quad \Sigma \frac{6!}{p!q!r!}(-2)^q(3)^4 x^p y^f z^r$$

Then, coefficient of xy^2z^3 is

$$p=1, q=2r=3$$

$$\Rightarrow \quad \frac{6!}{1!2!3!}(-2)^2(3)^3$$

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 2} \times 4 \times 27$$

$$= 240 \times 27 = 6480$$

Question7

The set of all real values of x for which the expansion of $\left(125x^2 - \frac{27}{x}\right)^{\frac{-2}{3}}$ is valid, is

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Options:

A. $\left(-\frac{3}{5}, \frac{3}{5}\right)$

B. $\left(-\infty, -\frac{3}{5}\right) \cup \left(\frac{3}{5}, \infty\right)$

C. $\left(-\frac{5}{3}, \frac{5}{3}\right)$

D. $\left(-\infty,-\frac{1}{3}\right) \cup\left(\frac{1}{3}, \infty\right)$

Answer: B

Solution:

$\left(125 x^2-\frac{27}{x}\right)^{-\frac{2}{3}}$

$\Rightarrow \quad 5^{-2}\left(x^2-\frac{27}{125 x}\right)^{-\frac{2}{3}}$

$\Rightarrow\left|\frac{27}{125 x^3}\right| \leq 1 \Rightarrow-1 \leq \frac{27}{125 x^3} \leq 1$

$\Rightarrow \quad-1 \leq\left(\frac{3}{5 x}\right)^3 \leq 1 \Rightarrow-1 \leq \frac{3}{5 x}<1$

$\Rightarrow \quad-\frac{5}{3}<\frac{1}{x}<\frac{5}{3} \Rightarrow \frac{1}{x}<\frac{5}{3}$

$\Rightarrow \quad \frac{3}{5}<x<\infty$

and $\quad \frac{1}{x}>-\frac{5}{3}$

$\Rightarrow \quad-\infty<x<-\frac{3}{5}$

$\therefore \quad x \in\left(-\infty,-\frac{3}{5}\right) \cup\left(\frac{3}{5}, \infty\right)$

Question8

If $3^{2n+2}-8n-9$ is divisible by $2^p, \forall n \in$ N, then the maximum value of P is

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Options:

A. 8

B. 7

C. 6

D. 9

Answer: C

Solution:

Given the expression $3^{2n+2} - 8n - 9$, we want to determine the maximum power of 2 that divides this expression for all $n \in \mathbb{N}$.

First, we can rewrite 3^{2n+2} as 9^{n+1}:

$9^{n+1} - 8n - 9$

Considering $9 = 1 + 8$, we can apply the binomial expansion to $(1+8)^{n+1}$:

$(1+8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1 \cdot 8 + {}^{n+1}C_2 \cdot 8^2 + {}^{n+1}C_3 \cdot 8^3 + \ldots$

Simplifying, we get:

$1 + 8(n+1) + 8^{2\,n+1}C_2 + {}^{n+1}C_3 \cdot 8^3 + \ldots$

Subtracting $8n + 9$ from this sequence, we have:

$1 + 8n + 8 + 8^{2\,n+1}C_2 + \ldots - 8n - 9$

Which simplifies to:

$8^2({}^{n+1}C_2 + 8^{n+1}C_3 + \ldots)$

The expression 8^2 can be further evaluated as 2^6. Thus, the binomial expansion shows that after simplification, the given expression is a multiple of 2^6.

Therefore, the maximum value of p such that the expression is divisible by 2^p is:

$\boxed{6}$

Question9

If the coefficient fo x^r in the expansion of $\left(1 + x + x^2 + x^3\right)^{100}$ is a_r and $S = \sum_{r=0}^{300} a_r$ then $\sum_{r=0}^{300} r \cdot a_r =$

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Options:

A. (50) S

B. (25)S

C. (150)S

D. (100)S

Answer: C

Solution:

Given the polynomial expression $(1+x+x^2+x^3)^{100}$, we are interested in the coefficient of x^r, denoted by a_r. To find the sum of the products $r \cdot a_r$, we perform the following steps:

First, evaluate the polynomial at $x=1$:

$(1+1+1^2+1^3)^{100} = (4)^{100}$

This shows that the sum of all coefficients, $\sum_{r=0}^{300} a_r$, equals 4^{100}.

Next, differentiate $(1+x+x^2+x^3)^{100}$ with respect to x:

$\frac{d}{dx}\left((1+x+x^2+x^3)^{100}\right) = 100(1+x+x^2+x^3)^{99}(1+2x+3x^2)$

Substitute $x=1$ into the derivative:

$$\begin{aligned}\sum_{r=0}^{300} r \cdot a_r &= 100(4)^{99}(1+2\cdot 1+3\cdot 1^2)\\ &= 100(4^{99})\cdot 6\end{aligned}$$

Simplify the expression to find:

$\sum_{r=0}^{300} r \cdot a_r = 600(4^{99}) = 150 \times 4^{100}$

Since $4^{100} = \sum_{r=0}^{300} a_r = S$, we have:

$\sum_{r=0}^{300} r \cdot a_r = 150S$

Thus, $\sum_{r=0}^{300} r \cdot a_r$ is equal to $150S$.

Question10

If $X \sim B(6,p)$ is a binomial variate and $\frac{P(X=4)}{P(X=2)} = \frac{1}{9}$, then $p =$

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Options:

A. $\frac{1}{2}$

B. $\frac{1}{9}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: D

Solution:

Given the binomial distribution $X \sim B(6, p)$ and the condition $\frac{P(X=4)}{P(X=2)} = \frac{1}{9}$, let's solve for p.

Start with the equation:

$9P(X = 4) = P(X = 2)$

For a binomial distribution:

$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$

Using this formula, we set up the probabilities:

$P(X = 4) = \binom{6}{4} p^4 (1 - p)^2$

$P(X = 2) = \binom{6}{2} p^2 (1 - p)^4$

We substitute these into our equation:

$9 \times \binom{6}{4} p^4 (1 - p)^2 = \binom{6}{2} p^2 (1 - p)^4$

Calculate the binomial coefficients:

$\binom{6}{4} = 15$

$\binom{6}{2} = 15$

Substitute these values:

$9 \times 15p^4 (1 - p)^2 = 15p^2 (1 - p)^4$

Cancel out the common term 15:

$9p^4 (1 - p)^2 = p^2 (1 - p)^4$

Divide both sides by $p^2 (1 - p)^2$ (assuming $p \neq 0$ and $p \neq 1$):

$9p^2 = (1 - p)^2$

Take the square root of both sides:

$3p = 1 - p$

Solve for p:

$3p + p = 1$

$4p = 1$

$p = \frac{1}{4}$

Therefore, the value of p is $\frac{1}{4}$.

Question11

If p and q are the real numbers such that the 7 th term in the expansion of $\left(\frac{5}{p^3}-\frac{3q}{7}\right)^8$ is 700 , then $49p^2=$

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Options:

A. $4q^2$

B. $9q^2$

C. $16q^2$

D. $25q^2$

Answer: B

Solution:

We know that $T_{r+1}={}^nC_r x^{n-r}a^r$

$$T_{6+1}={}^8C_6\left(\frac{5}{p^3}\right)^{8-6}\left(-\frac{3q}{7}\right)^6=700$$

$$=28\times\frac{25}{p^6}\times\frac{729q^6}{7^6}=700$$

$$\Rightarrow(7p)^6=\frac{28\times25\times729}{700}q^6$$

$$\Rightarrow(7p)^6=(3q)^6\Rightarrow 7p=\pm3q\Rightarrow 49p^2=9q^2$$

Question12

If T_4 represents the 4 th term in the expansion of $\left(5x+\frac{7}{x}\right)^{\frac{-3}{2}}$ and $x\notin\left[-\sqrt{\frac{7}{5}},\sqrt{\frac{7}{5}}\right]$, then $\left(x^7\sqrt{5x}\right)T_4=$

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Options:

A. $\frac{7^4}{2^5 5^3}$

B. $-\frac{7^4}{2^5 5^3}$

C. $-\frac{7^4}{2^4 5^3}$

D. $\frac{7^4}{2^4 5^3}$

Answer: C

Solution:

Given,

$\left(5x+\frac{7}{x}\right)^{-3/2}=(5x)^{-\frac{3}{2}}\left(1+\frac{7}{5x^2}\right)^{-3/2}$

We know that

$$(1+x)^{-n}=1-nx+\frac{n(n-1)}{2!}x^2$$
$$-\frac{n(n-1)(n-2)x^3}{3!}+\ldots$$

$\therefore T_4=\frac{1}{5x\sqrt{5x}}\left[\frac{-3}{2}\times\frac{5}{2}\times\frac{7}{2}\times\frac{1}{6}\times\left(\frac{7}{5x^2}\right)^3\right]$

$$\left(x^7\sqrt{5x}\right)T_4=-\frac{1}{5}\times\frac{3}{2}\times\frac{5}{2}\times\frac{7}{2}\times\frac{1}{6}\times\frac{7^3}{5^3}$$
$$=-\frac{7^4}{2^4 5^3}$$

Question13

If the coefficients of 3 consecutive terms in the expansion of $(1+x)^{23}$ are in arithmetic progression, then those terms are

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Options:

A. T_{10}, T_{11}, T_{12}

B. T_8, T_9, T_{10}

C. T_{13}, T_{14}, T_{15}

D. T_{14}, T_{15}, T_{16}

Answer: D

Solution:

Let 3 consecutive terms in the expansion of $(1+x)^{23}$ be

T_{r+1}, T_{r+2} and T_{r+3}

Coefficient of $T_{r+1} = {}^{23}C_r$

Coefficient of $T_{r+2} = {}^{23}C_{r+1}$

Coefficient of $T_{r+3} = {}^{23}C_{r+2}$

According to the question,

$${}^{23}C_{r+1} - {}^{23}C_r \quad = {}^{23}C_{r+2} - {}^{23}C_{r+1}$$

$$\Rightarrow \quad \frac{23!}{(r+1)!(22-r)!} - \frac{23!}{r!(23-r)!}$$

$$= \frac{23!}{(r+2)!(21-r)!} - \frac{23!}{(r+1)!(22-r)!}$$

$$\Rightarrow \quad \frac{23-r}{(r+1)!(23-r)!} - \frac{(r+1)}{(r+1)!(23-r)!}$$

$$= \frac{22-r}{(r+2)!(22-r)!} - \frac{r+2}{(r+2)!(22-r)!}$$

$$\Rightarrow \quad (r+2)[(23-r)-(r+1)]$$

$$\Rightarrow \quad = (23-r)[(22-r)-(r+2)]$$

$$\Rightarrow \quad (r+2)(22-2r) = (23-r)(20-2r)$$

$$\Rightarrow \quad r^2 - 84r + 416 = 0$$

$$\Rightarrow \quad (r-8)(r-13) = 0$$

$$\Rightarrow \quad r = 8, 13$$

Hence, 3 consecutive terms are either T_9, T_{10} and T_{11} or T_{14}, T_{15} and T_{16}.

Question14

The numerically greatest term in the expansion of $(3x - 16y)^{15}$, when $x = \frac{2}{3}$ and $y = \frac{3}{2}$, is

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Options:

A. 13th term

B. 14 th term

C. 15 th term

D. 16 th term

Answer: C

Solution:

Let the numerically greatest term in the expansion of $(3x - 16y)^{15}$ when $x = \frac{2}{3}$ and $y = \frac{3}{2}$ is T_{r+1}

$$T_{r+1} = {}^{15}C_r\left(3\left(\frac{2}{3}\right)\right)^{15-r}\left(-16\left(\frac{3}{2}\right)\right)^r$$

$$= {}^{15}C_r(2)^{15-r}(-24)^r$$

Clearly, the value the expression will be maximum only when the value r is largest possible even integer.

So, $r = 14 \quad [\because 0 \leq r \leq 15 \text{ and } r \in Z]$

$\therefore$ 15 th term will be the numerically greatest term.

Question15

For $n \in N$ the largest positive integer that divides $81^n + 20n - 1$ is k. If S is the sum of all positive divisors of k, then $S - k =$

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Options:

A. 117

B. 130

C. 115

D. 127

Answer: A

Solution:

To find largest positive integer K, that divides $81^n + 20n - 1$ for $n \in N$, we start evaluating the expression for small values of n

For $n = 1 : 81^1 + 20.1 - 1 = 100$

For $n = 2 : 81^2 + 20.2 - 1 = 6600$

Now,

The greatest common divisor of 100 and 6600 is 100 . Hence, $K = 100$ is the largest integer that divides

$81^n + 20n - 1$ for all $n \in N$

and the divisior of 100 are

$1, 2, 4, 5, 10, 20, 25, 50, 100$

$\Rightarrow S = 1 + 2 + 4 + 5 + 10 + 20 + 25 + 50 + 100 = 217$

Hence, $S - K = 117$

Question16

The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is

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Options:

A. 32

B. 33

C. 34

D. 35

Answer: B

Solution:

To determine the number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$, consider the general term in the expansion:

$T_{r+1}=\binom{256}{r}(\sqrt{3})^{256-r}(\sqrt[8]{5})^r$

This expression simplifies to:

$T_{r+1}=\binom{256}{r}(3)^{\frac{256-r}{2}}(5)^{\frac{r}{8}}$

For the term to be an integer, both $\frac{256-r}{2}$ and $\frac{r}{8}$ must be integers. Therefore:

$256-r$ must be even, implying r must be even.

$\frac{r}{8}$ also needs to be an integer, meaning r must be a multiple of 8.

Thus, r can be 0, 8, 16, 24, …, up to 256. This sequence forms an arithmetic progression (AP) where $r=0,8,16,\ldots,256$.

To find the total number of terms in this sequence:

The first term (a) is 0.

The common difference (d) is 8.

The last term (l) is 256.

Using the formula for the nth term of an AP, $l=a+(n-1)\cdot d$:

$256=0+(n-1)\cdot 8$

Solving for n:

$256=8(n-1)$

$n-1=\frac{256}{8}=32$

$n=32+1=33$

Therefore, the number of integral terms in the expansion is 33.

Question17

The expansion of $\left(1+x+x^2\right)^{-3/2}$ in powers of x is valid, if

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Options:

A. $|x| < 1$

B. $|x| < \frac{1}{2}$

C. $\left|x + \frac{1}{2}\right| < \frac{\sqrt{5}}{2}$

D.

$-\frac{1}{2} - \frac{\sqrt{5}}{2} < x < 1$

Answer: C

Solution:

We know that the expansion $(1+x)^n$ in power of x is valid if $|x| < 1$, where n is a rational number.

Now, $1 + x + x^2 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} - 1 + 1$$
$$= \left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 1$$

Now, the expansion of $\left(1 + x + x^2\right)^{-\frac{3}{2}}$ in power of x is valid, if

$$\left|\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right| < 1$$
$$\Rightarrow -1 < \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} < 1$$
$$\Rightarrow -\frac{3}{4} < \left(x + \frac{1}{2}\right)^2 < \frac{5}{4}$$
$$\Rightarrow 0 \leq \left(x + \frac{1}{2}\right)^2 < \frac{5}{4}$$
$$\left[\because \left(x + \frac{1}{2}\right)^2 \geq 0 \forall x \in R\right]$$
$$\Rightarrow \left|x + \frac{1}{2}\right| < \frac{\sqrt{5}}{2}$$

Question18

If $(1+x)^n = c_0 + c_1 x + c_2 x^2 + \ldots\ldots + c_n x^n$ for $n \in N$, then $c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \ldots\ldots + \frac{c_n}{n+1} =$

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Options:

A. $\frac{2^n-1}{n+1}$

B. $\frac{2^n-1}{n}$

C. $\frac{2^{n+1}-1}{n+1}$

D. $\frac{2^{n+1}-1}{n}$

Answer: A

Solution:

$(1+x)^n = c_0 + c_1 x + c_2 x^2 + \ldots c_n x^n.$

By integrating on both sides, we get

$$\frac{(1+x)^{n+1}}{n+1} = c_0 x + \frac{c_1 x^2}{2} + \frac{c_2 x^3}{3} + \frac{c_n x^{n+1}}{n+1}$$
$$\frac{2^{n+1}}{n+1} = c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \ldots \frac{c_n}{n+1}$$
$$\therefore \quad c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \ldots \frac{c_n}{n+1} = \frac{2^{n+1}}{n+1}$$

Question19

If the term independent of x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405 , then $k =$

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Options:

A. ± 1

B. 0

C. ± 3

D. ± 5

Answer: C

Solution:

$$\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$$

$$T_{r+1}={}^{10}C_r\left(x^{\frac{1}{2}}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$={}^{10}C_r\frac{x^{\frac{10-r}{2}}}{x^{2r}}(-k)^r$$

$$={}^{10}C_r x^{\frac{10-r}{2}-2r}(-k)^r$$

$$={}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$$

To be independent of $x, r=2$

$\therefore \quad 405={}^{10}C_2(-k)^2$

$\Rightarrow \quad 405=\frac{10\times 9}{2}(-k)^2$

$\Rightarrow \quad 405=45k^2$

$\Rightarrow \quad 9=k^2$

$\therefore \quad k=\pm 3$

Question20

The number of rational terms in the binomial expansion $(\sqrt[4]{5}+\sqrt[5]{4})^{100}$ is

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Options:

A. 10

B. 20

C. 6

D. 5

Answer: C

Solution:

To determine the number of rational terms in the binomial expansion $(\sqrt[4]{5}+\sqrt[5]{4})^{100}$, consider the term:

$$\sum_{r=0}^{100}\binom{100}{r}\cdot 5^{\frac{100-r}{4}}\cdot 4^{\frac{r}{5}}$$

For a term to be rational, both $\frac{r}{5}$ and $\frac{100-r}{4}$ must be integers.

Condition 1: $\frac{r}{5}$ must be an integer. This implies that r must be a multiple of 5. The possible values of r are:

$0, 5, 10, 15, 20, \ldots, 100.$

Condition 2: $\frac{100-r}{4}$ must also be an integer. For this condition, check which values from the list above also satisfy this.

For $r=0$, $\frac{100-0}{4}=25$ (integer)

For $r=20$, $\frac{100-20}{4}=20$ (integer)

For $r=40$, $\frac{100-40}{4}=15$ (integer)

For $r=60$, $\frac{100-60}{4}=10$ (integer)

For $r=80$, $\frac{100-80}{4}=5$ (integer)

For $r=100$, $\frac{100-100}{4}=0$ (integer)

The values of r that make both $\frac{r}{5}$ and $\frac{100-r}{4}$ integers are $0, 20, 40, 60, 80, 100$.

Therefore, there are 6 rational terms in the expansion.

Question21

The coefficient of x^{50} in the expansion of $(1+x)^{101}\left(1-x+x^2\right)^{100}$ is

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Options:

A. 0

B. -1

C. 50

D. 100

Answer: A

Solution:

Here, $(1+x)^{101}\left(1-x+x^2\right)^{100}$

$$
\begin{aligned}
&= (1+x)(1+x)^{100}\left(1-x+x^2\right)^{100} \\
&= (1+x)\left(1+x^3\right)^{100} \\
&= \left(1+x^3\right)^{100} + x\left(1+x^3\right)^{100} \\
&= \Sigma^{100}C_r x^{3r} + \Sigma^{100}C_r x^{3r+1}
\end{aligned}
$$

$3r = 50; 3r+1 = 50$

$r = \frac{50}{3}; \quad r = \frac{49}{3}$

$\Rightarrow$ Coefficient of $x^{50} = 0$
